Key

Math 4
U3 L3 I2 Activity Part 1

Name______________________

Date________

Learning goal:
I can identify important characteristics (asymptotes, holes, intercepts, and end behavior) of rational functions

From your previous work in mathematics, answer the following question:

1. If you are given a function $f(x)$, how do you find the following:

a. x-intercepts Substitute 0 for $f(x)$ (Plug in 0 for y). $f(x)=0$

Numerator = 0

b. y-intercept Substitute 0 for X... $x=0$

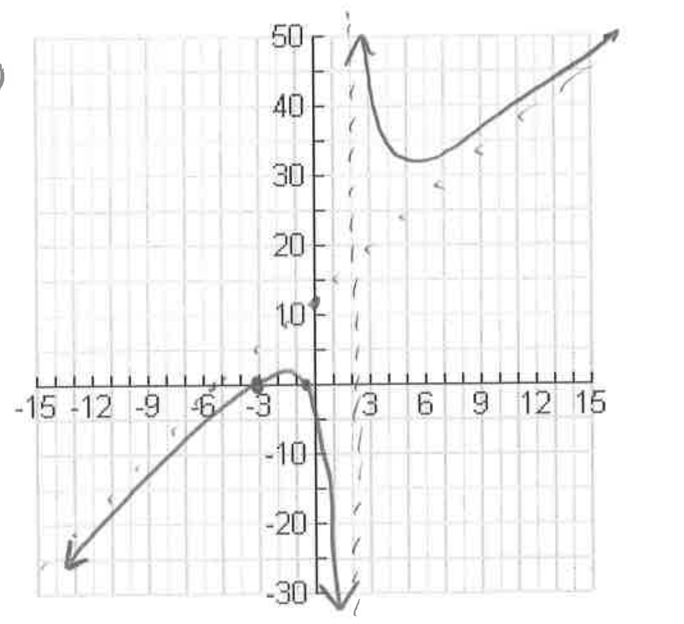
c. asymptotes Vert: find X-values that make only denominator = 0.

Horiz: End behavior! Imagine plugging in HUGE #s for x.

Oblique (Diagonal): Perform the division, only if $\frac{\text{Bigger Degree}}{\text{Smaller Degree}}$

2. Use the information above to algebraically find the x-intercepts, y-intercepts, and asymptotes of the rational functions below. Then graph the functions on your calculator to verify that you are correct. Sketch a graph on the provided axes.

a. $f(x)=\dfrac{2x^2+7x+3}{x-2} = \dfrac{(2x+1)(x+3)}{x-2}$



x-intercepts: $(-\frac{1}{2}, 0)$ & $(-3, 0)$

y-intercept: $(0, -\frac{3}{2})$

asymptotes:

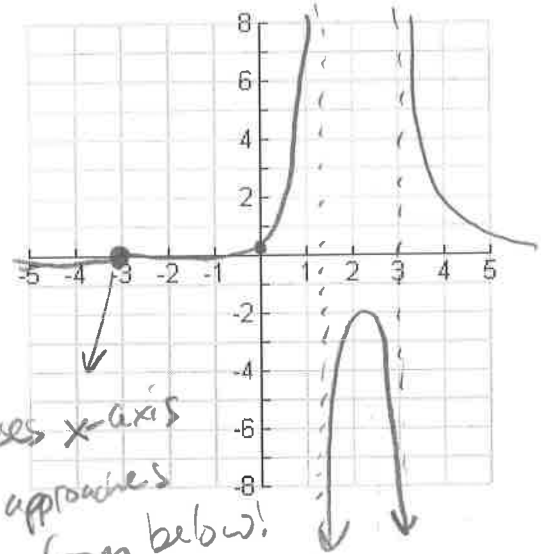
VeA: $x=2$

Horiz: None

Oblique:
$$\begin{array}{c|ccc} 2 & 2 & 7 & 3 \\ & & 4 & 22 \\ \hline & 2 & 11 & 25 \end{array}$$
$25 \rightarrow$ ignore remainder for oblique asymptotes.

$\boxed{y = 2x + 11}$

b. $g(x)=\dfrac{x+3}{3x^2-13x+12} = \dfrac{x+3}{(3x-4)(x-3)}$



x-intercepts: $(-3,0)$

y-intercept: $(0,\frac{1}{4})$

asymptotes:

Vert: $x=\frac{4}{3}$ & $x=3$

Horiz: $y=0$

Oblique: None

Crosses x-axis then approaches $y=0$ from below!

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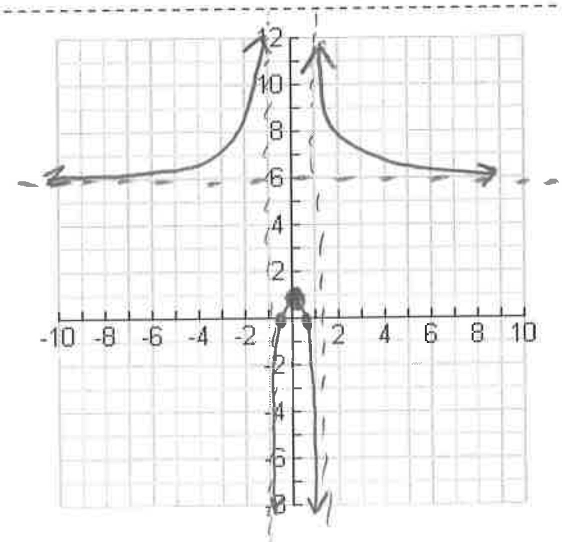
c. $h(x)=\dfrac{6x^2-x-1}{x^2-1} = \dfrac{(2x-1)(3x+1)}{(x+1)(x-1)}$



x-intercepts: $(\frac{1}{2},0)$ & $(-\frac{1}{3},0)$

y-intercept: $(0,1)$

asymptotes:

Vert: $x=1$ & $x=-1$

Horiz: $y=6$

Oblique: None

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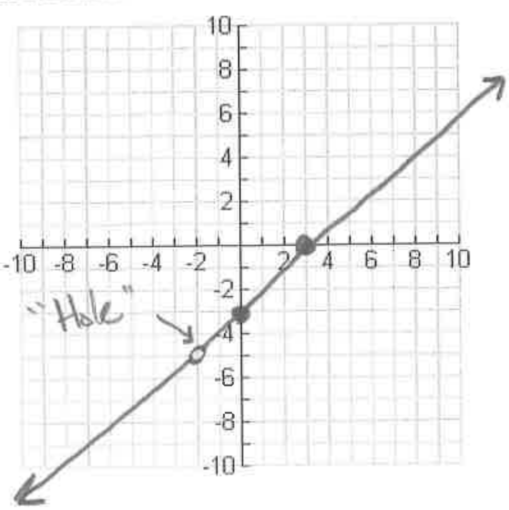
d. $k(x)=\dfrac{x^2-x-6}{x+2} = \dfrac{(x-3)\cancel{(x+2)}}{\cancel{x+2}} = x-3$

Linear!



x-intercepts: $(3,0)$

y-intercept: $(0,-3)$

asymptotes:

Vert: None

Horiz: None

Oblique: None

$k(-2)$

Hole: $(-2,-5)$

2. Based on your work above, fill in the table below:

Rational functions: $r(x)=\dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials

Asymptotes

	How do I know there is one?	***How do I find the equation?***
Vertical	There is an x-value that makes only the denominator $=0$, not the numerator + denominator.	Set denom. $=0$ + solve for x. Make sure that x-value doesn't also make numerator $=0$.
Horizontal	Only exist if degree of numerator is $\leq$ degree of denominator.	End behavior! Imagine plugging in HUGE #s for x. If degree numerator $<$ degree of denominator, equation is $y=0$.
Oblique	Only if degree of numerator $>$ degree of denominator by one.	Divide numerator by denominator + ignore remainder.

3. Explain when a graph will have a hole (also called a **removable discontinuity**) instead of a vertical asymptote (also called an **essential discontinuity**).

This occurs at x-values that make both the numerator + denominator $=0$. Will always occur at the factors you cancel out in the numerator + denominator.

4. Find the information for the following rational functions. Use your calculator to help, if necessary.

a. $f(x) = \dfrac{3x^2 - 8x - 16}{x - 5} = \dfrac{(3x+4)(x-4)}{x-5}$

Oblique

$$\begin{array}{r|rrr} 5 & 3 & -8 & -16 \\ & & 15 & 21 \\ \hline & 3 & 7 & \end{array}$$

x-intercept(s): $(-\frac{4}{3}, 0)$ + $(4, 0)$

y-intercept: $(0, \frac{16}{5})$

vert. asymptotes: $x = 5$

horiz. asymptotes: None

oblique asymptote: $y = 3x + 7$

Domain: $\{x : x \neq 5\}$

Hole: None

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b. $g(x) = \dfrac{2x^2 - 3x - 2}{6x^2 - 5x - 14} = \dfrac{(2x+1)\cancel{(x-2)}}{(6x+7)\cancel{(x-2)}} = \dfrac{2x+1}{6x+7}$

x-intercept(s): $(-\frac{1}{2}, 0)$

y-intercept: $(0, \frac{1}{7})$

vert. asymptotes: $x = -\frac{7}{6}$

horiz. asymptotes: $y = \frac{2}{6} = \boxed{\frac{1}{3}}$

oblique asymptote: None

Domain: $\{x : x \neq -\frac{7}{6}, x \neq 2\}$

Hole: $(2, \frac{5}{19})$

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c. $h(x) = \dfrac{x-1}{x^2 + 11x - 12} = \dfrac{\cancel{x-1}}{(x+12)\cancel{(x-1)}} = \dfrac{1}{x+12}$

x-intercept(s): None

y-intercept: $(0, \frac{1}{12})$

vert. asymptotes: $x = -12$

horiz. asymptotes: $y = 0$

oblique asymptote: None

Domain: $\{x : x \neq -12; x \neq 1\}$

Hole: $(1, \frac{1}{13})$

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d. $k(x) = \dfrac{4}{x^2 - 4x - 32} = \dfrac{4}{(x-8)(x+4)}$

$-\frac{4}{32} =$

x-intercept(s): None

y-intercept: $(0, -\frac{1}{8})$

vert. asymptotes: $x = 8$ + $x = -4$

horiz. asymptotes: $y = 0$

oblique asymptote: None

Domain: $\{x : x \neq -4, x \neq 8\}$

Hole: None